

1. a) The binding of the first acetylcholine molecule increases the open-to-closed ratio by a factor of $(1.2 \cdot 10^{-3}) : (5 \cdot 10^{-6}) = 240$, the binding of the second increases it by a factor of $(14) : (1.2 \cdot 10^{-3}) = 11667$.
- b) The free-energy contributions are 14 kJ/mol and 23 kJ/mol, respectively.

2. a) From Nernst's Equation, we have:

$$V = \frac{RT}{zF} \ln \left(\frac{C_{\text{extracellular}}}{C_{\text{intracellular}}} \right)$$

$$V_{\text{Na}^+} = 152.60 \text{ mV}$$

$$V_{\text{K}^+} = -243.00 \text{ mV}$$

$$V_{\text{Cl}^-} = -197.96 \text{ mV (note that for Cl}^- \text{ ion } z = -1)$$

b) Using the *Goldman-Hodgkin-Katz* equation, $V_M = -177.43 \text{ mV}$

c) $V_{\text{Na}^+} + V_{\text{K}^+} + V_{\text{Cl}^-} = -288.36 \text{ mV}$ This is not the same as b) because to calculate the resting membrane potential, the permeability of the ions also has to be taken into account.

3. a) If we consider the electric field across the membrane uniform, its magnitude E is given by:

$$E = -\frac{\Delta\phi}{d}$$

where $\Delta\phi$ is the potential difference between the plates and d is the distance separating the plates.

Using the membrane potential of -70 mV and separation distance of 4.7 nm, we get

$$E = -\frac{-70 \text{ mV}}{4.7 \text{ nm}} = 14.89 \text{ mV/nm} = 1.5 \times 10^5 \text{ V/cm}$$

If you applied an electric field of this magnitude to two metal electrodes separated by a 1-cm air gap, they would ionize the air between the electrodes.

b) Using the Nernst equation: $V_m = \frac{RT}{zF} \ln \left(\frac{C_{\text{extracellular}}}{C_{\text{intracellular}}} \right)$

$$\frac{C_{\text{extracellular}}}{C_{\text{intracellular}}} = e^{V_m \frac{zF}{RT}} = e^{-0.070 \left(\frac{1 \cdot 96485}{8.314 \cdot 310} \right)} = 0.07276 = \frac{1}{13.74}$$

4. a) Given that

$$\frac{dn}{dt} = \alpha_n(V)(1-n) - \beta_n(V)n$$

Initially, with $V_m = 0$, the state variable n has a steadystate value (i.e., when $dn/dt = 0$) given by:

$$n_{\infty}(0) = \frac{\alpha_n(0)}{\alpha_n(0) + \beta_n(0)}$$

When V_m is clamped to a new level V_c , the gating variable n will eventually reach a new steady state value given by:

$$n_{\infty}(V_c) = \frac{\alpha_n(V_c)}{\alpha_n(V_c) + \beta_n(V_c)}$$

The solution to the first equation that satisfies these two boundary conditions is a simple exponential of the form:

$$n(t) = n_{\infty}(V_c) - (n_{\infty}(V_c) - n_0(0))e^{-t/\tau_n(V_c)}$$

This equation describes the time course of n in response to a step change in command voltage

b) The original values from the paper are:

$$\alpha_n = 0.01 (V + 10) / \left(\exp \frac{V + 10}{10} - 1 \right),$$

$$\beta_n = 0.125 \exp (V/80),$$

$$\alpha_m = 0.1 (V + 25) / \left(\exp \frac{V + 25}{10} - 1 \right),$$

$$\beta_m = 4 \exp (V/18),$$

$$\alpha_h = 0.07 \exp (V/20),$$

$$\beta_h = 1 / \left(\exp \frac{V + 30}{10} + 1 \right).$$

Rate Constants	A	B	C	D	F	H
α_n	0.1	0.01	-1	10	10	1
β_n	0.125	0	0	0	80	1
α_m	2.5	0.1	-1	25	10	1
β_m	4	0	0	0	18	1
α_h	0.07	0	0	0	20	1
β_h	1	0	1	30	10	1