

Homework is due at the beginning of Lecture (8:15AM) on Thursday, February 6<sup>th</sup>, 2020.

1. The ratio of open to closed forms of the acetylcholine receptor channel containing zero, one, and two bound acetylcholine molecules is  $5 \cdot 10^{-6}$ ,  $1.2 \cdot 10^{-3}$ , and 14, respectively.

a) By what factor is the open-to-closed ratio increased by the binding of the first/second acetylcholine molecule?  
b) What are the corresponding free-energy contributions to channel opening at 25 °C?

2.

Ions	C <sub>a</sub> (mM)	C <sub>i</sub> (mM)
Na <sup>+</sup>	110	9.2
K <sup>+</sup>	2.5	130
Cl <sup>-</sup>	100	4

a) Using the Nernst equation, calculate the individual potentials of the ions given in the table above. Assume  $RT/F = 61.5\text{mV}$ .  
b) To account for multiple ions, the Nernst equation has to be modified and the result is given by the *Goldman-Hodgkin-Katz* equation:

$$V_M = \frac{RT}{F} \cdot \ln \frac{P_{\text{Na}} \cdot [\text{Na}^+]_a + P_{\text{K}} \cdot [\text{K}^+]_a + P_{\text{Cl}} \cdot [\text{Cl}^-]_i}{P_{\text{Na}} \cdot [\text{Na}^+]_i + P_{\text{K}} \cdot [\text{K}^+]_i + P_{\text{Cl}} \cdot [\text{Cl}^-]_a}$$

For a typical neuron at rest,  $P_{\text{K}}:P_{\text{Na}}:P_{\text{Cl}} = 1:0.05:0.45$ . Calculate the potential of nerve cell membrane.

c) Is the value obtained from *Goldman-Hodgkin-Katz* equation equal to sum of individual ion potentials? Why or why not?

3. The resting membrane potential of a cell is about -70 mV at 37°C, and the thickness of a lipid bilayer is about 4.7 nm.  
a) What is the strength of the electric field across the membrane in V/cm? What do you suppose would happen if you applied this voltage to two metal electrodes separated by a 1-cm air gap?  
b) What is the equilibrium concentration ratio of extracellular to intracellular K<sup>+</sup>?

4. Voltage clamp experiments, similar to the original Hodgkin and Huxley studies of squid giant axon were conducted, wherein the membrane potential starts in the resting state ( $V_m = 0$ ) and is then instantaneously stepped to a new clamp voltage  $V_c$ .

- a) Initially, with  $V_m = 0$ , the state variable  $n$  has a steady-state value (i.e., when  $dn/dt = 0$ ) and when  $V_m$  is clamped to a new level  $V_c$ , the gating variable  $n$  will eventually reach a new steady-state value. Recall that the differential equation governing the state variable  $n$  is given by:

$$\frac{dn}{dt} = \alpha_n(V)(1-n) - \beta_n(V)n$$

What is the time course of the state variable  $n$  that controls gating of the  $K^+$  channel, under these circumstances?

- b) In neural simulation software packages, the rate constants in Hodgkin Huxley-style models are often parameterized using a generic functional form:

$$\beta(V); \alpha(V) = \frac{A + BV}{C + H \exp\left(\frac{V + D}{F}\right)}$$

What values of the constants (A, B, etc.) should be substituted in the software package to obtain solutions for  $\alpha_n$ ,  $\beta_n$ ,  $\alpha_m$ ,  $\beta_m$ ,  $\alpha_h$ ,  $\beta_h$  as determined in the original Hodgkin and Huxley studies. Refer to their original paper for the actual values - *J. Physiol.* (1952) 117, 500-544