Homework should be completed by the ***beginning*** of Lecture (8:15AM) on Thursday, December 3rd, 2020.

1. Natural selection is a powerful force in evolution because cells with even a small growth advantage quickly outgrow their competitors. Consider a cell culture that contains 1 million bacterial cells that double every 20 minutes. A single cell in this culture acquires a mutation that allows it to divide faster, with a generation time of only 15 minutes. Assuming that there is an unlimited food supply and no cell death, how long would it take before the progeny of the mutated cell became predominant in the culture? (Before calculating the result, make a guess: would it take a day, week, month or year?) How many cells are present at this time?

Note: The number of cells *N(t)* in the culture at time *t* is described by the equation:

$$N(t) = N_0 \times 2^{t/G},$$

where N_0 is the number of cells at time zero and *G* is the generation time.

2. Diffraction Limited Optics

   a) For a resolution of 200 nm with 488 nm laser light, what N.A. does the objective need to have?

   b) If the laser beam does not completely fill the back aperture of the objective, the effective N.A. is decreased. If the back aperture of the above objective is 8 mm in diameter and the laser beam is 2 mm in diameter, what is the diffraction-limited spot size in this configuration?

3. What percentage of photons emitted isotropically from a particle at the focus of a 1.49 N.A. oil-immersion objective is collected and transmitted through the objective? Assume a transmission coefficient for the objective lenses of 85 %, and an index of refraction of water of 1.33 and an index of refraction of the immersion oil of 1.52.

   What would be the N.A. of an air objective with the same collection efficiency?

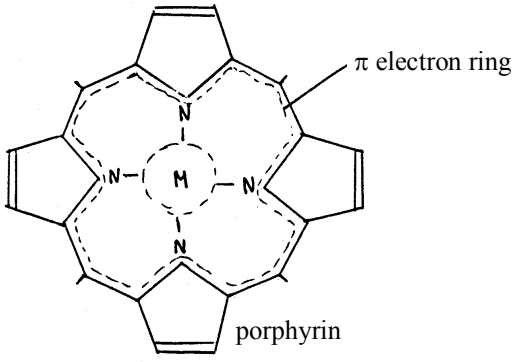
4. A compound microscope uses a very short focal length objective lens to form a greatly enlarged image. This image is then viewed with a short focal length eyepiece used as a simple magnifying glass.
An object is placed at distance o from the objective of a compound microscope (very near to the focus of the objective). The lenses are distance l apart. Assuming that the focal length of objective lens is f_o and that of the eyepiece is f_e, perform the following tasks:
   a) Draw a ray diagram showing the working of this compound microscope.
   b) Derive the formula for magnification of objective using the lens formula and given parameters
   c) Derive the formula for overall magnification assuming the eyepiece acts as a simple magnifying lens.
   d) Describe the Intermediate and Final images: Are they Real/Virtual, Diminished/Magnified, Upright/Inverted?
   e) Calculate the overall magnification when the object is placed at a distance of 50 mm from the objective with focal length of 40 mm. The eyepiece has a focal length of 50 mm and the lenses are 240 mm apart.

5. Soret Absorption Band
   a) Find the expression for the energy levels of a porphyrin molecule by treating the π-electrons in the inner ring as free electrons. For a one dimensional free electron gas, the Schrödinger equation is given by:

$$-\frac{\hbar^2}{2m_e}\frac{d^2\Psi}{d\,s^2} = E\Psi,$$

   subject to the boundary condition:

$$\Psi(s) = \Psi(s+L).$$



   b) Assuming a circumference of L = 2.2 nm and 18 electrons in the ring, calculate the wavelength of light absorption by the electronic transition from the ground state to the lowest level excited state.

6. For a protein labeled in two positions with a FRET pair, A FRET efficiency of 80 % has been measured. The Förster distance, R_0, of this FRET pair has been determined to be 59 Å assuming a κ^2 of 2/3. From anisotropy measurements, it is known that the acceptor is flexible and the donor is rigid. Hence, κ^2 could theoretically lie anywhere between 1/3 and 4/3. What are the minimum and maximum possible separations of the donor-acceptor dye pair?